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RECONSTRUCTION OF EFFECTIVE TRANSFER COEFFICIENTS FOR THE RADIATIVE-CONDUCTIVE HEAT TRANSFER PROBLEM

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Numerical results are presented for the coefficients in the nonlinear inverse radiative-conductive heat transfer problem.

Calculation of temperature fields in absorbing, emitting, and scattering media necessitates joint solution of the heat conduction and radiation transfer equations [1]. The complexity of the initial set of integrodifferential equations as well as the absence of reliable data on optical and thermophysical properties necessitates use of various approximate methods for solution of this class of problems.

In the present work we give results of numerical solution of a simplified mathematical model consisting of a nonlinear heat conduction equation with effective thermophysical coefficients that take account of energy transfer by conduction and radiation in the semitransparent material under consideration.

The effective coefficients are found by solving the one-dimensional inverse problem. Additional information on the temperature distribution in time at some points of the specimen $u(x_k, t)$, $k = \overline{1, M}$, $x_k \in (0, L)$, $t_1 \le t \le t_2$ is found from solution of the direct radiative-conductive heat transfer problem.

The medium is assumed to be absorbing and radiating, gray; the boundaries, diffusely radiating and reflecting; the plane-parallel case is considered.

With the assumptions made, the radiative-conductive heat transfer problem can be expressed by the following set of integrodifferential equations [1]:

$$C\rho \frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial U}{\partial x} - 2\pi \int_{-1}^{1} I\mu'\, d\mu'\right), \tag{1}$$

$$(x, t) \in \Omega = \{(x, t) \mid 0 < x < L,\ t_1 < t \le t_2\},$$

$$U(x, 0) = \varphi(x), \quad 0 \le x \le L, \tag{2}$$

$$-\lambda \frac{\partial U}{\partial x}\Big|_{x=0} = q_1(t), \quad t_1 \le t \le t_2, \tag{3}$$

$$-\lambda \frac{\partial U}{\partial x}\Big|_{x=L} = q_2(t), \quad t_1 \le t \le t_2, \tag{4}$$

$$\mu \frac{\partial I}{\partial x} + \beta I = \beta I_p(U), \quad 0 < x < L, \tag{5}$$

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$$I(0, \mu) = \varepsilon_1 I_p(U)|_{x=0} + 2R_1 \int_0^1 I(0, \mu')\mu' \, d\mu', \quad \mu > 0, \tag{6}$$

$$I(L, \mu) = \varepsilon_2 I_p(U)|_{x=L} + 2R_2 \int_0^1 I(L, \mu')\mu' \, d\mu', \quad \mu < 0, \tag{7}$$

$$I_p(U) = \frac{n^2 \sigma U^4}{\pi}.$$

The problem of determining the thermophysical and optical properties of the material from rigorous solution of inverse problem (1)-(7) is rather complex since even the direct problem in a simplified formulation requires cumbersome computations and substantial machine time. Therefore, in what follows the process considered is assumed to be described approximately by the following model:

$$C_{\text{ef}}(\tilde{U}) \frac{\partial \tilde{U}}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_{\text{ef}}(\tilde{U}) \frac{\partial \tilde{U}}{\partial x}\right), \quad (x, t) \in \Omega, \tag{8}$$

$$\tilde{U}(x, 0) = \varphi(x), \quad 0 \le x \le L, \tag{9}$$

$$-\lambda_{\text{ef}}(\tilde{U}) \frac{\partial U}{\partial x}\Big|_{x=0} = q_1(t), \quad t_1 \le t \le t_2, \tag{10}$$

$$-\lambda_{\text{ef}}(\tilde{U}) \frac{\partial \tilde{U}}{\partial x}\Big|_{x=L} = q_2(t), \quad t_1 \le t \le t_2. \tag{11}$$

The effective coefficients $C_{\text{ef}}(\tilde{U})$ and $\lambda_{\text{ef}}(\tilde{U})$ are found from the condition of the minimum residue functional

$$J = \sum_{k=1}^{M} \int_{t_1}^{t_2} [U(x_k, t) - \tilde{U}(x_k, t)]^2 \, dt, \tag{12}$$

where $U(x_k, t)$ and $\tilde{U}(x_k, t)$ are the temperatures at the point x_k at time t, obtained from solution of problems (1)-(7) and (8)-(11), respectively.

The problem of finding minimum functional (12) is solved in the finite-dimensional space R^m, consisting of vectors $\bar{a}$ whose elements are parameters of the piecewise-linear approximation of the unknown coefficients $C_{\text{ef}}(\tilde{U})$ and $\lambda_{\text{ef}}(\tilde{U})$. The vector $\bar{a}^*$, giving a minimum for functional (12), is found by the conjugate gradient method. Iteration regularization [2] is used. As a criterion characterizing uniqueness and the level of conditionality of the problem, we use a quantity equal to the ratio of the variance of the vector of parameters $\bar{a}$ to the temperature variance at the observation points and calculated by the formula [3]

$$S = \left(\sqrt{\left(\operatorname{diag}\left(\frac{\partial \tilde{U}^T}{\partial \bar{a}} \frac{\partial \tilde{U}}{\partial \bar{a}^T}\right)\right)}\right)^{-1}. \tag{13}$$

The problem posed was solved numerically. As parameters for the model problem, the following optical and thermophysical properties of optical glass were used [4]: $\rho = 2720$ kg/m^3; $\lambda = 0.73$ W/(m·deg); $C = 795$ J/(kg·deg); $L = 10^{-2}$ m; $R_i = 0.8$; $\varepsilon_i = 0.2$; $i = 1, 2$; $\beta = 50$ m^{-1}. The boundary conditions: $\varphi(x) = 293$ K, $q_1(t) = 0$, $q_2(t) = 2 \cdot 10^4$ W/m^2.

Numerical experiments were carried out for temperatures ranging from 980 to 1300 K. For refinement of the structure of model (8)-(11), three variants of the inverse problem were considered:

TABLE 1. Results of Comparison of the Variants by the Functional Value, Conditionality Level, and Number of Iterations

Variant	J	S	N
1	$5.5 \cdot 10^4$	$1.2 \cdot 10^{-4}$	17
2	$4.5 \cdot 10^4$	$2.8 \cdot 10^{-4}$	10
3	$5.2 \cdot 10^4$	$1.08 \cdot 10^{-4}$	6

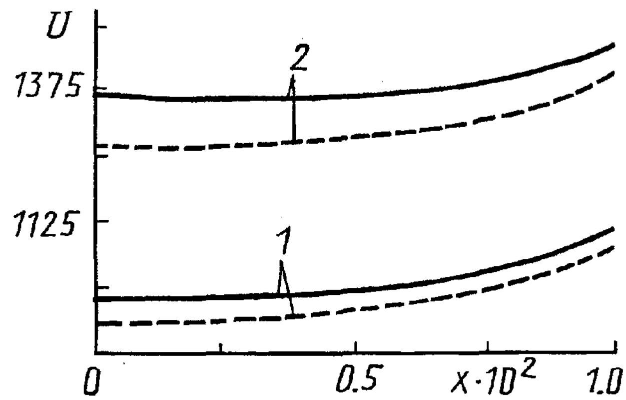

Fig. 1. Temperature distribution U (K) over the coordinate $x \cdot 10^2$ (m) at different times: 1) $t = 740$, 2) 1080 sec; solid lines show numerical solution of problem (8)-(11); dashed lines refer to numerical solution problem (1)-(7) with radiation energy transfer neglected.

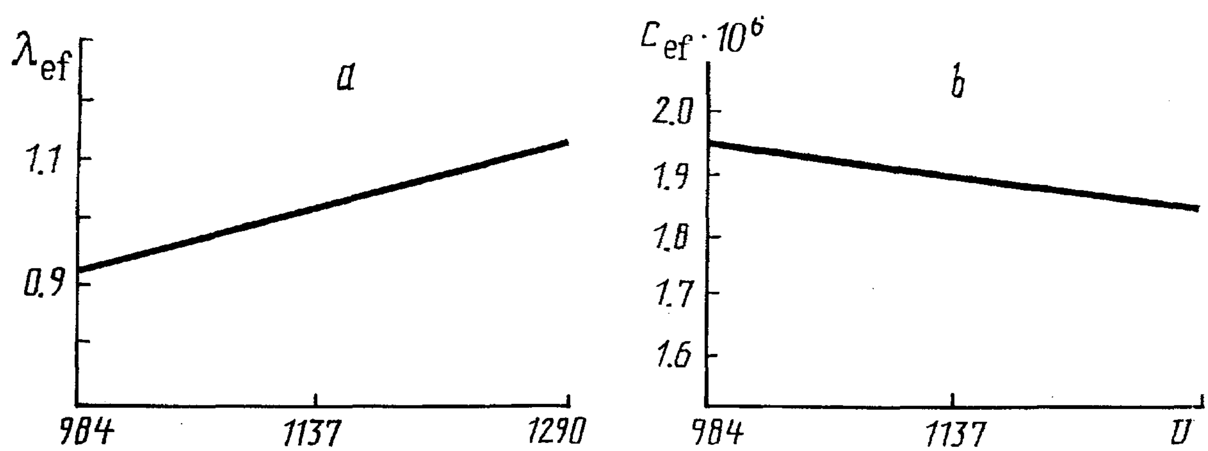

Fig. 2. Plot of the effective thermal conductivity λ_{ef} (W/(m·deg)) (a) and heat capacity C_{ef} (J/m^3·deg) (b) versus the temperature U (K).

1) the coefficients $C_{ef}(\tilde{U})$ and $\lambda_{ef}(\tilde{U})$ were reconstructed jointly;

2) $C_{ef}(\tilde{U})$ was reconstructed at fixed $\lambda_{ef}(\tilde{U})$, equal to the thermal conductivity λ;

3) λ_{ef} was reconstructed under the condition $C_{ef} = C\rho$.

In Table 1 three variants of the inverse problem are compared by the value of the functional J, the conditionality level S, and the iteration number N. It can be seen from Table 1 that the best approximation for the functional is given by variant 2; however, the parameters are sensitive to input data errors and the convergence to the solution is worse than for the other variants. Variant 1 is optimal in accuracy and stability of the solution in the case where the effective thermal conductivity and heat capacity are reconstructed simultaneously. For this variant these coefficients were determined within 8%. The difference in the distribution of the temperature calculated from the solution of initial problem (1)-(7) and simplified problem (8)-(11) did not exceed 6-7%.

Results of numerical calculations of unsteady-state temperature fields and effective thermophysical properties are given in Figs. 1-3. Figure 1 shows the effect of radiative energy transfer on the temperature distribution in the material. Predicted effective thermophysical coefficients are given in Figs. 2 and 3.

The present numerical experiments have shown that effective thermophysical coefficients, found by solving the inverse heat conduction problem, can be used with satisfactory accuracy for solution of one-dimensional radiative-conductive heat transfer problems in the formulation (1)-(7).

NOTATION

U, absolute temperature, K; C, specific heat, J/(kg·K); ρ, density, kg/m^3; λ, thermal conductivity, W/(m·deg); I, integral radiation intensity, W/(m^3·ster); μ, cosine of the angle between the chosen coordinate direction and the radiation direction; β, integral absorptivity, m^{-1}; ε_i, integral radiation coefficient, $i = 1, 2$; R_i, integral reflection index; n, refractivity; σ, Stefan-Boltzmann constant; λ_{ef}, effective thermal conductivity, W/(m·deg); C_{ef}, effective heat capacity, J/(m^3·K); q, heat flux density, W/m^2.

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